<table>
<thead>
<tr>
<th>Day</th>
<th>Topic</th>
<th>Homework/Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fri.</td>
<td>11.1 Angular Momentum Quiz 10</td>
<td>RE 11.a; HW10: 13*, 21, 30, “39”</td>
</tr>
<tr>
<td>Mon.</td>
<td>11.2-.3, (.12) Rotational + Translational</td>
<td>RE 11.b EP10</td>
</tr>
<tr>
<td>Tues.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mon.</td>
<td>11.4-.6, (.13) Angular Momentum &amp; Torque</td>
<td>RE 11.c EP11</td>
</tr>
<tr>
<td>Tues.</td>
<td>11.7 - .9, (.11) Torque</td>
<td>RE 11.d</td>
</tr>
<tr>
<td>Wed.</td>
<td>L11 Rotation Course Evals</td>
<td></td>
</tr>
<tr>
<td>Fri.</td>
<td>11.10 Quantization, Quiz 11</td>
<td>RE 11.e</td>
</tr>
<tr>
<td>Mon.</td>
<td>Review for Final (1-11)</td>
<td>HW11: Ch 11 Pr’s 39, 57, 64, 74, 78 &amp; Practice Exam</td>
</tr>
</tbody>
</table>
Introducing Angular Momentum

The measure of motion \textit{about} a point

\textbf{Magnitude}

\[ |L| = |p_{\text{around}}||\mathbf{r}_{\text{sun}\rightarrow\text{Earth}}| = p||\mathbf{r}_{\text{sun}\rightarrow\text{Earth}}||\sin \theta \]

Only 'around' component of momentum counts

\[ p_{\text{around}} = p \cos(90^\circ - \theta) = p \sin \theta \]

\[ r_\perp = r_{\text{sun}\rightarrow\text{Earth}} \sin \theta \]

\[ |L| = |p_{\text{around}}||\mathbf{r}_{\text{sun}\rightarrow\text{Earth}}| = p||\mathbf{r}_{\text{sun}\rightarrow\text{Earth}}||\sin \theta = p||r_\perp| \]
Using Angular Momentum
The measure of motion about a point

Magnitude

\[ |L| = |p_{around}|r = |p||r| = |p||r| \sin \theta \]

What is the magnitude of the angular momentum about location \( K \), for the object shown below? The magnitude of the object's momentum \( |p| = 7 \text{ kg} \cdot \text{m/s} \), the distance \( |r| = 0.6 \text{ m} \), and the angle \( \theta = 150^\circ \).
Using Angular Momentum

The measure of motion about a point

Magnitude

\[ |\vec{L}| = |p_{\text{around}}||\vec{r}| = |p||\vec{r}_\perp| = |p||\vec{r}| \sin \theta \]

Determine the magnitude of the translational angular momentum of the particle at location \( O \) relative to each point: \( A, B, C, D, E, F, G, \) and \( H \).

\[ |\vec{L}_F| = \]

\[ |\vec{L}_G| = \]

\[ |\vec{L}_H| = \]

\[ |\vec{L}_A| = \]

\[ |\vec{L}_B| = \]

\[ |\vec{L}_C| = \]

\[ |\vec{L}_D| = \]

\[ |\vec{L}_E| = \]

\[ |\vec{p}| = 50 \text{ kg} \cdot \text{m/s} \]

\( h = 12 \text{ m} \)

\( b = 9 \text{ m} \)

\( w = 11 \text{ m} \)
Using Angular Momentum

The measure of motion about a point

Direction

Distinguish with Right Hand Rule

Orient Right hand so fingers curl with motion, then thump points in conventional direction of angular momentum

The one direction momentum and position vectors never point is $z$-Axis of rotation

But that's also true for $\vec{p}$
Using Angular Momentum
The measure of motion about a point
Direction
Example

Distinguish with Right Hand Rule
Orient Right hand so fingers curl with motion, then thump points in conventional direction of angular momentum

What are the directions of Angular Momentum for particle 1 about point A and particle 2 about point A

a)  

b)  

c)  

d)  

(tip of z-axis arrow pointing at you)
A comet orbits the Sun, in the xy plane. Its momentum is shown by the red arrow. What is the direction of the comet's angular momentum about the Sun?

1) +x  
2) –x  
3) +y  
4) –y  
5) +z  
6) –z  
7) toward the sun  
8) away from the sun
Using Angular Momentum
The measure of motion about a point

Direction
Distinguish with Right Hand Rule

Determine the direction of the translational angular momentum of the particle at location $O$ relative to each point: $A$, $B$, $C$, $D$, $E$, $F$, $G$, and $H$.

\[ \hat{L}_F = \]
\[ \hat{L}_G = \]
\[ \hat{L}_H = \]

\[ \hat{L}_A = \]
\[ \hat{L}_B = \]
\[ \hat{L}_C = \]
\[ \hat{L}_D = \]
\[ \frac{|p|}{50 \text{ kg} \cdot \text{m/s}} \]
A ball falls straight down in the $xy$ plane. Its momentum is shown by the red arrow. What is the direction of the ball's **angular momentum** about location $A$?

1) $+x$
2) $-x$
3) $+y$
4) $-y$
5) $+z$
6) $-z$
7) zero magnitude

Given these values, what is the magnitude of the ball’s angular momentum about $A$?

1) 10 kg m$^2$/s
3) 40 kg m$^2$/s
5) 0
Using Angular Momentum

The measure of motion *about* a point

Magnitude *and* Direction

\[ \mathbf{\hat{x}} \]
\[ \mathbf{\hat{y}} \]
\[ \mathbf{\hat{z}} \]

= 

Cross Product

Similarly for position and momentum in the y-z 

and for position and momentum in the x-z
Using Angular Momentum

The measure of motion about a point

Magnitude and Direction

\[ \vec{r} = \frac{\vec{p}}{m} \]

Example: say you have a mass that, at some instant, has linear momentum \( \vec{p} = \langle 4, 2, 0 \rangle \text{kg} \cdot \text{m/s} \) and is \( \vec{r}_A = \langle 5, 3, 0 \rangle \text{m} \) from some point A. What is its angular momentum about this point?
What is the direction of \(<0, 0, 3> \times <0, 4, 0>\)?

1) $+x$
2) $-x$
3) $+y$
4) $-y$
5) $+z$
6) $-z$
7) zero magnitude

What is the direction of \(<0, 4, 0> \times <0, 0, 3>\)?

What is the direction of \(<0, 0, 6> \times <0, 0, -3>\)?
If an object is traveling at a constant speed in a vertical circle, how does the object's angular momentum change as the object goes from the top of the circle to the bottom of the circle?

1. $|\vec{r}|$ increases
2. $|\vec{r}|$ decreases
3. $|\vec{r}|$ stays the same but the direction of $\vec{r}$ changes
4. The direction and magnitude of $\vec{r}$ remain the same
Using Angular Momentum
The measure of motion about a point
Effect of a radial force (like gravity or electric)

\[
\frac{d}{dt} \mathbf{L}_{E-S} = \frac{d}{dt} \left| \mathbf{r}_{E-S} \times \mathbf{p}_E \right| = \frac{d\mathbf{r}_{E-S}}{dt} \times \mathbf{p}_E + \mathbf{r}_{E-S} \times \frac{d\mathbf{p}_E}{dt}
\]

\[
\frac{d}{dt} \mathbf{L}_{E-S} = \mathbf{v}_E \times \mathbf{p}_E + \mathbf{r}_{E-S} \times \mathbf{F}_{Earth\leftarrow sun} = 0
\]

Parallel Parallel

\[
\mathbf{L}_{E-S} = \text{constant}
\]
Kepler and Planetary Orbits:

Sweeping our equal area in equal time: If $\Delta t_1 = \Delta t_2$, then $A_1 = A_2$

Some Geometric manipulation

![Geometric Diagram](image)

Some mathematical manipulation…

$$A_2 = \frac{1}{2} \frac{\Delta t_2}{m} m v r \sin \theta = \frac{1}{2} \frac{\Delta t_2}{m} p r \sin \theta = \frac{1}{2} \frac{\Delta t_2}{m} |\vec{p} \times \vec{r}| = \frac{1}{2} \frac{\Delta t_2}{m} |\vec{L}|$$

Since $L$ is constant (and $m$ is constant), $A$ is the same for the same time interval
Relating Energy, Radius and Angular Momentum in Circular Orbit

Angular Momentum: \( L_{\text{orbit}} = rp = rmv \) (r & p perpendicular)

Kinetic and Gravitational Potential Energy: \( E = K + U \)

Kinetic energy: \( K = \frac{1}{2} mv^2 = \frac{L^2_{\text{orbit}}}{2mr^2} \)

Potential energy: \( U = -G \frac{Mm}{r} \)

Gravitational Force and Circular motion:

\[
\left| F_{\text{net}} \right| = m \frac{v^2}{r}
\]

\[
-\frac{U}{r} = G \frac{Mm}{r^2} = \frac{2}{r} K
\]

\[
-U = 2K \quad \Rightarrow K + U = -K
\]

\[
G \frac{Mm}{r} = 2 \frac{L^2_{\text{orbit}}}{2mr^2} \quad K + U = -\frac{L^2_{\text{orbit}}}{4mr^2}
\]

\[
r = \frac{L^2}{mGMm}
\]

Later in chapter will apply same reasoning to electric interaction
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<td></td>
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<td></td>
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<td>RE 11.d</td>
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