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Collisions
Short, Sharp Shocks

Smack!
Which of the following is a property of all collisions - both “elastic” and “inelastic” collisions?

(1) The internal energy of the system after the collision is different from what it was before the collision.
(2) The total momentum of the system doesn’t change.
(3) The total kinetic energy of the system doesn’t change.
Which of the following is a property of all “elastic” collisions?

(1) The colliding objects interact through springs.
(2) The kinetic energy of one of the objects doesn’t change.
(3) The total kinetic energy is constant at all times -- before, during, and after the collision.
(4) The total kinetic energy after the collision is equal to the total kinetic energy before the collision.
(5) The elastic spring energy after the collision is greater than the elastic spring energy before the collision.
Example

System = carts A & B

Often know initial motion, want to predict final motion

Two unknowns: $\vec{v}_{A,f}$ and $\vec{v}_{B,f}$

Generally need two equations to solve for them

True for all collisions

$$\Delta \vec{p}_{A&B} = \Delta \vec{p}_A + \Delta \vec{p}_B \approx 0$$

$$\Delta E_{A&B} = \Delta E_A + \Delta E_B + \Delta U_{A&B} \approx 0$$

$$\Delta K_A + \Delta E_{\text{int.} A} \quad \Delta K_B + \Delta E_{\text{int.} B}$$
1-D Collision

Special Case: “Maximally Inelastic” – hit & stick

System = carts A & B

\[ \Delta \vec{p}_A + \Delta \vec{p}_B \approx 0 \]

\[ (m_A \vec{v}_{A.f} - m_A \vec{v}_{A.i}) + (m_B \vec{v}_{B.f} - m_B \vec{v}_{B.i}) \approx 0 \]

\[ (m_A \vec{v}_{f} - m_A \vec{v}_{A.i}) + (m_B \vec{v}_{f} - m_B \vec{v}_{B.i}) \approx 0 \]

\[ \vec{v}_f \approx \frac{m_A \vec{v}_{A.i} + m_B \vec{v}_{B.i}}{m_A + m_B} \]
1-D Collision

Special Case: Perfectly Elastic (all internal changes ‘bounce back’)

System = carts A & B

\[ \Delta E_{A&B} = \Delta K_A + \Delta E_{A,\text{int}} + \Delta K_B + \Delta E_{B,\text{int}} + \Delta U_{A&B} \]

\[
\left( \frac{1}{2} m_A v_{A,f}^2 - \frac{1}{2} m_A v_{A,i}^2 \right) + \left( \frac{1}{2} m_B v_{B,f}^2 - \frac{1}{2} m_B v_{B,i}^2 \right) \approx 0 \quad (v' s \ll c)
\]

**Equation 1**

\[
\left( \frac{p_{A,f}^2}{2m_A} - \frac{p_{A,i}^2}{2m_A} \right) + \left( \frac{p_{B,f}^2}{2m_B} - \frac{p_{B,i}^2}{2m_B} \right) \approx 0
\]

**Equation 2**

\[
\vec{p}_{A,f} + \vec{p}_{B,f} = \vec{p}_{A,i} + \vec{p}_{B,i}
\]
1-D Collision

**Special Case: Perfectly Elastic** (all internal changes ‘bounce back’)

Extra special case: Say B is initially stationary

\[
\left( \frac{m_A - m_B}{m_B + m_A} \right) \vec{v}_{A,i} = \vec{v}_{A,f} \\
\vec{P}_{A,f} = \frac{2m_A}{(m_B + m_A)} \vec{v}_{A,i}
\]

\[
\vec{v}_{B,f} = \frac{2m_A}{(m_B + m_A)} \vec{v}_{A,i}
\]

Initially Stationary

Finally

\[
\vec{P}_{B,f}
\]

\[
2\vec{v}_{A,i}, \quad \vec{v}_{A,f}, \quad \vec{v}_{B,f}
\]

\[
\vec{v}_{A,i}, \quad \vec{v}_{A,i}, \quad \vec{v}_{A,i}, \quad \vec{v}_{A,i}, \quad \vec{v}_{A,i}, \quad \vec{v}_{A,i}
\]

\[
0, \quad 1
\]
1-D Collision

Special Case: Perfectly Elastic
Transform for B initially moving

Say I’m watching this collision while I’m just sitting here. According to *me*

- $\vec{v}_{A.i.me}$
- $\vec{v}_{B.i.me} = 0$

Initially I see

Say you’re watching this collision while walking by with velocity $\vec{v}_{you}$. Relative to *you* the initial velocities are

- $\vec{v}_{A.i.you} = \vec{v}_{A.i.me} - \vec{v}_{you}$
- $\vec{v}_{B.i.you} = -\vec{v}_{you}$

Initially you see

Similarly, after the collision, according to *me*

And according to *you*

- $\vec{v}_{A.f.you} = \vec{v}_{A.f.me} - \vec{v}_{you}$
- $\vec{v}_{B.f.you} = \vec{v}_{B.f.me} - \vec{v}_{you}$

Finally I see

Finally you see
1-D Collision

Special Case: Perfectly Elastic Transform for B initially moving

So, completely rephrasing things from your perspective:

Initially you see

\[ \vec{v}_{A.i.you} = \vec{v}_{A.i.me} - \vec{v}_{you} \]

\[ \vec{v}_{B.i.you} = -\vec{v}_{you} \]

Finally I see

\[ \vec{v}_{A.f.you} = \vec{v}_{A.f.me} - \vec{v}_{you} \]

\[ \vec{v}_{B.f.you} = \vec{v}_{B.f.me} - \vec{v}_{you} \]
2-D Collision: Scattering

Slow (v<<c)

\[ \vec{p}_{p,i} \]

\[ \vec{p}_{T,i} = 0 \]

\[ m_p \]

\[ m_T \]
2-D Collision: Scattering

Slow ($v << C$)

\[ \vec{p}_{p.f} = \left\langle p_{p.f} \cos \theta_p, p_{p.f} \sin \theta_p, 0 \right\rangle \]

\[ \vec{p}_{p.i} = \left\langle p_{p.i}, 0, 0 \right\rangle \]

\[ \vec{p}_{T.i} = 0 \]

\[ m_p \]

\[ m_T \]

Three Equations / Four Unknowns

Need another equation

Or more information

Momentum Principle

\[ \vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0 \]

\[ \hat{x} : p_{p.f} \cos \theta_p + p_{T.f} \cos \theta_T - p_{p.i} = 0 \]

\[ \hat{y} : p_{p.f} \sin \theta_p - p_{T.f} \sin \theta_T - 0 = 0 \]

Energy Principle

\[ \left( E_{p.f} + E_{T.f} \right) - \left( E_{p.i} + E_{T.i} \right) = 0 \]

\[ K_{p.f} + K_{T.f} - K_{p.i} + \Delta E_{\text{int.p}} + \Delta E_{\text{int.t}} + \Delta U_{t,p} = 0 \]

\[ \frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} - \frac{p_{p.i}^2}{2m_p} = 0 \]

if Elastic
2-D Collision: Scattering

**Example.** Say a 2kg puck moving at 3m/s collides with a 4kg puck that’s just sitting there. Say the 2kg puck travels off at 30° and the 4kg one travels down at 50°. What are the final speeds and is the collision elastic?

\[
\vec{p}_{p.f} = \langle p_{p.f} \cos \theta_p, p_{p.f} \sin \theta_p, 0 \rangle
\]

Momentum Principle

\[
\begin{align*}
\hat{x} : & \quad p_{p.f} \cos \theta_p + p_{T.f} \cos \theta_T - p_{p.i} = 0 \\
\hat{y} : & \quad p_{p.f} \sin \theta_p - p_{T.f} |\sin \theta_T| - 0 = 0
\end{align*}
\]

Energy Principle – if Elastic

\[
\frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} - \frac{p_{p.i}^2}{2m_p} = 0
\]
2-D Collision: Scattering

Slow (v<<C), Elastic Collision

\[ \vec{p}_{p.f} = \left( p_{p.f} \cos \theta_p , p_{p.f} \sin \theta_p , 0 \right) \]

Three Equations / Four Unknowns

Need another equation

Special case: equal masses

\[ m_p = m_T \]

\[ \frac{p_{p,i}^2}{2m_p} = \frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} \]

\[ \Rightarrow p_{p,i} = p_{p.f} + p_{T.f} \]

\[ p_{p.i}^2 = \vec{p}_{p_i} \cdot \vec{p}_{p_i} = \left( \vec{p}_{p.f} + \vec{p}_{T.f} \right) \cdot \left( \vec{p}_{p.f} + \vec{p}_{T.f} \right) \]

\[ p_{p,i} = \vec{p}_{p.f} + \vec{p}_{T.f} \]

\[ p_{p.i}^2 = p_{p.f}^2 + p_{T.f}^2 + 2 \vec{p}_{p.f} \cdot \vec{p}_{T.f} \]

compare

A: projectile missed target

B: \[ \theta_p + |\theta_T| = 90^\circ \]

Must be zero
2-D Collision: Scattering

Slow (v<<C), Elastic Collision

\[ \vec{p}_{p.f} = \begin{pmatrix} p_{p.f} \cos \theta_p, \ p_{p.f} \sin \theta_p, 0 \end{pmatrix} \]

Three Equations / Four Unknowns

Need another equation

Momentum Principle

\[ \vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0 \]

\[ \hat{x} : p_{p.f} \cos \theta_p + p_{T.f} \cos \theta_T - p_{p.i} = 0 \]

\[ \hat{y} : p_{p.f} \sin \theta_p - p_{T.f} \sin \theta_T - 0 = 0 \]

Energy Principle – if Elastic

\[ \left( E_{p.f} + E_{T.f} \right) - \left( E_{p.i} + E_{T.i} \right) = 0 \]

\[ K_{p.f} + K_{T.f} - K_{p.i} = 0 \]

\[ \frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} - \frac{p_{p.i}^2}{2m_p} = 0 \]
2-D Collision: Scattering

Slow (v<<C), Elastic Collision

\[ \vec{p}_{p,f} = \langle p_{p,f} \cos \theta_p, p_{p,f} \sin \theta_p, 0 \rangle \]

\[ \vec{p}_{p,i} = \langle p_{p,i}, 0, 0 \rangle \]

\[ F = \frac{d\vec{p}}{dt} \]

Component of momentum along line of contact changes, other component remains constant

\( b = \text{Impact Parameter} \)

Momentum Principle

\[ \vec{p}_{p,f} + \vec{p}_{T,f} - \vec{p}_{p,i} = 0 \]

\( \hat{x} : p_{p,f} \cos \theta_p + p_{T,f} \cos \theta_T - p_{p,i} = 0 \)

\( \hat{y} : p_{p,f} \sin \theta_p - p_{T,f} |\sin \theta_T| - 0 = 0 \)

Three Equations / Four Unknowns

Need another equation

Collision geometry and Impact Parameter

Energy Principle – if Elastic

\[ (E_{p,f} + E_{T,f}) - (E_{p,i} + E_{T,i}) = 0 \]

\[ K_{p,f} + K_{T,f} - K_{p,i} = 0 \]

\[ \frac{p_{p,f}^2}{2m_p} + \frac{p_{T,f}^2}{2m_T} - \frac{p_{p,i}^2}{2m_p} = 0 \]
Which of these is true?

1) The larger the impact parameter, the larger the scattering angle (deflection).
2) The larger the impact parameter, the smaller the scattering angle (deflection).
2-D Collision: Scattering
Qualitative effect of Impact Parameter

Miss: $b$ too big

Glancing Blow: $b$ big

Solid Blow: $b$ small

No deflection

minor deflection

major deflection
Probability and Cross-sectional Area

*Playing Darts Badly*
2-D Collision: Scattering – Discovering Nucleus

Diagram showing a 2-D collision experiment with a radioactive source emitting particles, some of which are scattered by a gold foil and detected on a ZnS screen.

Graph below the diagram shows the fraction of scatters versus the scattering angle $\theta$. The distribution appears to be exponential with a peak at small angles and a tail extending to larger angles.
In the Rutherford experiment, what was surprising to the experimenters?

(1) Sometimes the alpha “rays” passed right through the gold foil.
(2) Sometimes the alpha “rays” were deflected slightly when they passed through the gold foil.
(3) Sometimes the alpha “rays” bounced back from the gold foil.
Deeper Collisions

- relativistic speeds
- identifying ‘mystery’ particles
- the mathematical trick of analyzing in the center-of-mass reference frame

2-D Collision: Scattering

Fast

\[ \vec{p}_{p,i} = \langle p_{p,i}, 0, 0 \rangle \]

\( m_p \) \hspace{0.5cm} \( \vec{p}_{T,i} = 0 \)

\( m_T \)

Conservation of Momentum

\[ \vec{p}_{p,f} + \vec{p}_{T,f} - \vec{p}_{p,i} = 0 \]

\( \hat{x} \) : \( p_{p,f} \cos \theta_p + p_{T,f} \cos \theta_T - p_{p,i} = 0 \)

\( \hat{y} \) : \( p_{p,f} \sin \theta_p - p_{T,f} |\sin \theta_T| - 0 = 0 \)

where \[ \vec{p} = \frac{m \vec{v}}{\sqrt{1 - (\frac{v}{c})^2}} \]

Conservation of Energy

\[ (E_{p,f} + E_{T,f}) - (E_{p,i} + E_{T,i}) = 0 \]

where \[ E = \frac{mc^2}{\sqrt{1 - (\frac{v}{c})^2}} \]

\[ \sqrt{(p_{p,f}c)^2 + (m_{p,f}c^2)^2} + \sqrt{(p_{T,f}c)^2 + (m_{T,f}c^2)^2} - \sqrt{(p_{p,i}c)^2 + (m_{p,i}c^2)^2} - m_{T,i}c^2 = 0 \]

Note: initial and final masses differ for inelastic collisions
2-D Collision: Scattering
Fast

Note: initial and final masses differ for inelastic collisions

Recall: particle energy

\[ E_p = K_p + E_{p,\text{int}} \]

\[ E_{p,\text{int}} = m_p c^2 \]

Elastic

\[ \Delta E_{p,\text{int}} = \Delta m_p c^2 = 0 \]

\[ \sqrt{(p_{p,f}c)^2 + (m_p c^2)^2} + \sqrt{(p_{T,f}c)^2 + (m_T c^2)^2} - \sqrt{(p_{p,i}c)^2 + (m_p c^2)^2} - m_T c^2 = 0 \]

In-Elastic

\[ \Delta E_{p,\text{int}} = \Delta m_p c^2 \neq 0 \]

\[ \sqrt{(p_{p,f}c)^2 + (m_{p,f} c^2)^2} + \sqrt{(p_{T,f}c)^2 + (m_{T,f} c^2)^2} - \sqrt{(p_{p,i}c)^2 + (m_{p,i} c^2)^2} - m_{T,i} c^2 = 0 \]
2-D Collision: Fast

Example: A beam of high energy $\pi^-$ (negative pions) is shot at a flask of liquid hydrogen, and sometimes a pion interacts through the strong interaction with a proton in the hydrogen, the reaction is $\pi^- + p^+ \rightarrow \pi^- + X^+$ where $X^+$ is a positively charged particle of unknown mass (essentially an excited proton.)

A proton’s rest mass is 938 MeV, and a pion’s rest mass is 140 MeV. The incoming pion has momentum 3 GeV/c. It scatters through 40°, and its momentum drops to 1.5 GeV/c.

What is the rest mass of the $X^+$?

Conservation of Energy

Conservation of Momentum

\[
\hat{x}: p_{\pi, f} \cos \theta_{\pi} + p_{X, f} \cos \theta_X - p_{\pi, i} = 0 \quad \Rightarrow \quad p_{X, f} \cos \theta_X = p_{\pi, i} - p_{\pi, f} \cos \theta_{\pi}
\]

\[
\hat{y}: p_{\pi, f} \sin \theta_{\pi} - p_{X, f} |\sin \theta_X| = 0 \quad \Rightarrow \quad p_{X, f} |\sin \theta_X| = p_{\pi, f} \sin \theta_{\pi}
\]

\[
\left( p_{X, f} \cos \theta_X \right)^2 + \left( p_{X, f} |\sin \theta_X| \right)^2 = \left( p_{\pi, i} - p_{\pi, f} \cos \theta_{\pi} \right)^2 + \left( p_{\pi, f} \sin \theta_{\pi} \right)^2
\]

\[
p_{X, f} = \sqrt{\left( p_{\pi, i} - p_{\pi, f} \cos \theta_{\pi} \right)^2 + \left( p_{\pi, f} \sin \theta_{\pi} \right)^2}
\]

\[
p_{X, f} = \sqrt{\left( p_{\pi, i} \right)^2 + \left( p_{\pi, f} \right)^2 - 2 p_{\pi, i} p_{\pi, f} \cos \theta_{\pi}}
\]
2-D Collision: Fast

Example: A beam of high energy $\pi^-$ (negative pions) is shot at a flask of liquid hydrogen, and sometimes a pion interacts through the strong interaction with a proton in the hydrogen, the reaction is $\pi^- + p^+ \rightarrow \pi^- + X^+$ where $X^+$ is a positively charged particle of unknown mass (essentially an excited proton.)

A proton's rest mass is 938 MeV, and a pion's rest mass is 140 MeV. The incoming pion has momentum 3 GeV/c. It scatters through 40°, and its momentum drops to 1.51 GeV/c.

What is the rest mass of the $X^+$?

Conservation of Momentum

\[
p_{X.f} = \sqrt{(p_{\pi.i})^2 + (p_{\pi.f})^2 - 2p_{\pi.i}p_{\pi.f} \cos \theta_{\pi}}
\]

Conservation of Energy

\[
\sqrt{(p_{\pi.f}c)^2 + (m_{\pi.f}c^2)^2} + \sqrt{(p_{\pi.c})^2 + (m_{\pi.c})^2} - \sqrt{(p_{\pi.i}c)^2 + (m_{\pi.i}c^2)^2} = 0
\]

\[
\sqrt{(p_{\pi.c})^2 + (m_{\pi.c})^2} = \sqrt{(p_{\pi.i}c)^2 + (m_{\pi.i}c^2)^2} + m_{\pi.i}c^2 - \sqrt{(p_{\pi.f}c)^2 + (m_{\pi.f}c^2)^2}
\]

Could do algebraically further, or just plugin numbers and simplify
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Collisions
Short, Sharp Shocks

Smack!