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Ball-Spring Model

Molecule

Solid

F = -k(r - r_{eq})

U_{Morse} = E_M \left[1 - e^{-\alpha(r - r_{eq})}\right]^2 - E_M

Morse’ semi-empirical approximate

‘good enough’
Ball-Spring Model

Morse’ semi-empirical approximate

\[ U_{Morse} = E_M \left[ 1 - e^{-\alpha(r-r_{eq})} \right]^2 - E_M \]

Experimentally determining the constants

- \( E_M = U_{eq} \): Break molecule
- \( r_{eq} \): For solid, from density and atomic mass
- For molecule: rotational spectrum (more later)
- \( \alpha \): From vibrational spectrum (more now)
Ball-Spring Model

Finding $\alpha$: relating to mass-spring

Taylor Series Approximation

$$U(r) \approx U(r_{eq}) + \frac{dU}{dr} \bigg|_{r_{eq}} (r - r_{eq}) + \frac{1}{2} \frac{d^2U}{dr^2} \bigg|_{r_{eq}} (r - r_{eq})^2$$

$$|F| = -\left|\frac{dU}{dr}\right|$$

at equilibrium $= 0$

$$\frac{d^2U}{dr^2} = \frac{d|F|}{dr} = k$$

$$U(r) \approx U(r_{eq}) + \frac{1}{2} k(r - r_{eq})^2$$

$$\frac{d^2U_M}{dr^2} = 2E_M \alpha^2$$

Morse’ semi-empirical approximate

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$U_{Morse} = E_M \left[1 - e^{-\alpha(r-r_{eq})}\right]^2 - E_M$$
Ball-Spring Model

Potential Energy in Solid

- \[ U_{bond,1}(r) \approx U\left(r_{eq}\right) + \frac{1}{2}k\left(r_1 - r_{eq}\right)^2 \]

- \[ U_{solid}(r) \approx N_{bonds}U\left(r_{eq}\right) + \sum_{bonds} \frac{1}{2}k_{bond}s^2 \]

- \[ U_{solid}(r) \approx N_{bonds}U\left(r_{eq}\right) + \frac{1}{2}k_{total}s_{total}^2 \]

Recall: \[ k_{total} = \frac{Area}{Length \cdot r_{eq}}k_1 \]
Spring force and potential

System = weight + spring
Active environment = none

\[ \Delta E = W_{\text{system} \leftarrow \text{ext}} = 0 \]

\[ \Delta E_{W,S} = \Delta E_{\text{rest},W} + \Delta E_{\text{rest},S} + \Delta K_W + \Delta K_S + \Delta U_{W,S} = 0 \]

\[ U_{1,2} = \frac{1}{2} k_s s^2 + E_{eq} \]

\[ \frac{1}{2} m_w v_{w,f}^2 - \frac{1}{2} m_w v_{w,i}^2 + \left( \frac{1}{2} k_s s_f^2 \right) - \left( \frac{1}{2} k_s s_i^2 \right) = 0 \]

A horizontal spring with stiffness 3 N/m has a relaxed length of 25 cm (0.25 m). A mass of 50 grams (0.050 kg) is attached and you stretch the spring to a total length of 29 cm (0.29 m). The mass is then released from rest and moves with negligible friction. What is the kinetic energy of the mass at the moment when the spring returns through the position its equilibrium length, 25 cm?

1) 2.4e-3 J
2) 4.8e-3 J
3) 6e-2 J
4) 1.26e-1 J
5) 1.5 J
Horizontal Ball on spring

ball on spring energy force momentum.py
Consider a block falling onto a vertical spring.

States:
A: Block 0.8 m above floor
B: Block just touching top of spring
C: Block come to rest

To find the speed of the block just before it hits the spring, what should we pick for initial and final states?

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<td>3</td>
<td>A</td>
<td>C</td>
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<tr>
<td>4</td>
<td>C</td>
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Consider a block falling onto a vertical spring.

**States:**
- **A:** Block 0.8 m above floor
- **B:** Block just touching top of spring
- **C:** Block come to rest

To find the **maximum compression of the spring**, what should we pick for initial and final states?

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Ball-Spring Model
Kinetic Energy in Solid
Ball-Spring Model

Thermal Expansion

For higher energies, average bond length is greater

$\Delta L$ proportional to $\Delta E$

Thermometers:
$\Delta T$ proportional to $\Delta L$
$\Delta L$ proportional to $\Delta E$
$\Delta T$ proportional to $\Delta E$
Heat Capacity

$\Delta T$ proportional to $\Delta E_{\text{int}}$

$$\Delta E_{\text{int}} = C \Delta T$$

$C =$ Heat capacity

$C$ should be material specific and scale with amount of material

$$C = c_v m \quad c_v = \text{(mass) specific Heat capacity}$$

$$\Delta E_{\text{int}} = c_v m \Delta T$$

The thermal energy of the 1000 grams of water increased 7000 J. What was the temperature increase in Kelvins of the water? The heat capacity of water is $c_v = 4.2 \text{ J/K}$ on a per-gram basis.

1) 0.0006 K
2) 0.6 K
3) 1.7 K
4) 1667 K
5) Insufficient information
Say you do the historic falling-weight / paddle-wheel in bucket experiment. The mass drops, the wheel spins, and the 10 kg’s of water warm up. We find that the difference between the change in gravitational potential and change in kinetic energies is $40 \times 10^3$ J. A thermometer stuck in the water tells you that its temperature has risen $\Delta T = 0.96$ Kelvin. What’s the mass-specific heat capacity of water?
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**Force as negative gradient (3-D slope) of Potential Energy**

small change in potential

\[ dU_{1,2} = -\vec{F}_{1\to2} \cdot d\vec{r}_{1\to2} = -\left( F_{1\to2,x} \, dx + F_{1\to2,y} \, dy + F_{1\to2,z} \, dz \right) \]

so

\[ \vec{F}_{1\to2} = \left\langle F_{1\to2,x}, F_{1\to2,y}, F_{1\to2,z} \right\rangle = -\left\langle \frac{\partial U_{1,2}}{\partial x_{1\to2}}, \frac{dU_{1,2}}{dy_{1\to2}}, \frac{dU_{1,2}}{dz_{1\to2}} \right\rangle \]

Which of the following expressions satisfies the requirement that \( F_x = -dU_s / dx = -k_s x \) where \( x \) is measured from the equilibrium position (\( C \) is a constant)?

1) \( U_s = \frac{1}{2} k_s x^2 + C \)
2) \( U_s = -\frac{1}{2} k_s x^2 + C \)
3) \( U_s = k_s x^2 + C \)
4) \( U_s = k_s + C \)
5) \( U_s = -k_s + C \)
**Ball-Spring Model**

For the Ball-Spring Model, the force acting on each particle is given by:

\[ F = -k(r - r_{eq}) \]

The potential energy is given by:

\[ U = \frac{1}{2}k(r - r_{eq})^2 \]

Morse's semi-empirical approximate equation for the potential energy is:

\[ U_{Morse} = E_M \left[ 1 - e^{-\alpha(r - r_{eq})} \right]^2 - E_M \]

where \( E_M \) is the equilibrium energy and \( r_{eq} \) is the equilibrium distance.
Two lead bricks moving in the $+x$ and $-x$ directions, each with kinetic energy $K$, smash into each other and come to a stop. What happened to the energy?

1) The observable kinetic energy changed to thermal energy, a form of rest energy.
2) The total energy of the system decreased by an amount $2K$.
3) Since the blocks were moving in opposite directions, the initial kinetic energy of the system was zero, so there was no change in energy.