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Example Gravitational System: Earth and Sun

System = Earth + Sun
Active environment = none

\[ \Delta E = W_{\text{system} \rightarrow \text{ext}} = 0 \]

\[ \Delta E_{E,S} = \Delta E_{\text{rest,E}} + \Delta E_{\text{rest,S}} + \Delta K_E + \Delta K_S + \Delta U_{E,S} = 0 \]

\[ \Delta E_{E,S} = \Delta K_E + \Delta U_{E,S} = 0 \]

\[ K_{E,f} + U_{E,S,f} = K_{E,i} + U_{E,S,i} \]

\[ \frac{1}{2} m_E v_{E.f}^2 - G \frac{m_E m_S}{r_{ESf}} = \frac{1}{2} m_E v_{E.f}^2 - G \frac{m_E m_S}{r_{ESi}} \]
Which of the following graphs of $U$ vs $r$ represents the gravitational potential energy, $U = -\frac{GMm}{r}$?
Different Initial Speeds / kinetic Energies, Different Paths

(orbit noncircular, with energy vs position.py)
In which graph does the cyan line correctly represent the sum of kinetic energy plus potential energy?
Conceptual Understanding from Energy Diagrams
Ex. Nuclear Potential

Energy

Limits / Turning Points

K=0 K=0

K+U = Const
Energy Limits / Turning Points

\[ K = 0, \quad K = 0 \]

\[ K(r_1) + U(r_1) = \text{Const} \]

\[ K(r_1) = \text{Const} - U(r_1) \]
Conceptual Understanding from Energy Diagrams
Ex. Nuclear Potential

Energy

Limits / Turning Points

K(r₁) + U(r₁) = Const
K(r₁) = Const - U(r₁)
K + U = Const
Energy Limits / Turning Points

\[ K(r_1) + U(r_1) = \text{Const} = K(r_2) + U(r_2) \]

\[ K(r_1) = \text{Const} - U(r_1) \]
The system is a comet and a star. In which case(s) will the comet escape from the star and never return?

GRAVITY WELLS
Scaled to Earth Surface Gravity

This chart shows the "depth" of various solar system gravity wells.

Each well is scaled such that rising out of a physical well of that depth—in constant Earth surface gravity—would take the same energy as escaping from that planet's gravity in reality.

Each planet is shown cut in half at the bottom of its well, with the depth of the well measured down to the planet's surface.

The planet sizes are to the same scale as the wells, interplanetary distances are not to scale.

Depth = \( \frac{GM}{2R} \) + \( K \times \text{cubic term} \)

-=\( \text{cubic term} \)

\( M \) = planet's mass

\( R \) = planet's radius

\( K \) = Weber's constant

\( g \) = local gravitational acceleration

Jupiter is not much larger than Saturn, but much more massive at its size, adding more mass just makes it deeper due to the extra squeezing of gravity.

If you dropped a few dozen more Jupiters into it, the pressure would ignite fusion and make it a star.

It takes the same amount of energy to launch something on an escape trajectory away from Earth as it would to launch it 6600 m upward under constant 9.8 m/s² Earth gravity.

Hence, Earth's well is 6600 m deep.
Force as negative gradient (3-D slope) of Potential Energy

small change in potential

\[ dU_{1,2} = -\vec{F}_{1\rightarrow 2} \cdot d\vec{r}_{1\rightarrow 2} = -(F_{1\rightarrow 2,x} \, dx + F_{1\rightarrow 2,y} \, dy + F_{1\rightarrow 2,z} \, dz) \]

Say only moves in the x direction, then

\[ dU_{1,2} = -F_{1\rightarrow 2,x} \, dx \quad \text{so} \quad -\frac{dU_{1,2}}{dx} = F_{1\rightarrow 2,x} \]

Similarly, if only moves in the y direction, then

\[ dU_{1,2} = -F_{1\rightarrow 2,y} \, dy \quad \text{so} \quad -\frac{dU_{1,2}}{dy} = F_{1\rightarrow 2,y} \]

or, if only moves in the z direction, then

\[ dU_{1,2} = -F_{1\rightarrow 2,z} \, dz \quad \text{so} \quad -\frac{dU_{1,2}}{dz} = F_{1\rightarrow 2,z} \]

Moving in all directions,

\[ \vec{F}_{1\rightarrow 2} = \left< F_{1\rightarrow 2,x}, F_{1\rightarrow 2,y}, F_{1\rightarrow 2,z} \right> = -\left< \frac{\partial U_{1,2}}{\partial x_{1\rightarrow 2}}, \frac{dU_{1,2}}{dy_{1\rightarrow 2}}, \frac{dU_{1,2}}{dz_{1\rightarrow 2}} \right> \]
Gravitational Potential Energy

Newton's Universal Law of Gravitation

\[
\vec{F}_{2\leftarrow 1} = \frac{1}{4\pi\varepsilon_0} \frac{m_1 q_2}{r_{2\leftarrow 1}^2} \hat{r}_{2\leftarrow 1}
\]

\[
G = \frac{6.67\times 10^{-11} \text{ N} \cdot \text{m}^2}{(\text{kg})^2}
\]

Gravitational Potential Energy

\[
U_{1,2} = \frac{1}{4\pi\varepsilon_0} \frac{m_1 q_2}{r_{1\leftarrow 2}}
\]

\[
U_{1,2.electric} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{1\leftarrow 2}}
\]
Example: Ionize Hydrogen. In a hydrogen atom the electron averages around $10^{-10}$ m from the proton. When a hydrogen atom is ionized, the electron is stripped away. What is the change in electric potential energy when such an atom is ionized?

\[
U_{e,p\text{.}\text{electric}} \approx \frac{1}{4\pi\varepsilon_o} \frac{q_e q_p}{r_{e\leftarrow p}}
\]

System = electron + proton

Active environment = none

\[
\Delta U_{e,p\text{.}\text{elct}} = \frac{1}{4\pi\varepsilon_o} \left( \frac{-e^2}{r_f} - \frac{-e^2}{r_i} \right) = \frac{e^2}{4\pi\varepsilon_o} \left( \frac{1}{r_i} - \frac{1}{r_f} \right) \approx \infty
\]

\[
\Delta U_{e,p\text{.}\text{elct}} = 2.3 \times 10^{-18} \, J
\]

Or in eV’s (divide by electron charge)

\[
= 2.3 \times 10^{-18} \, J \times \frac{1e}{1.6 \times 10^{-19} \, C} = 14 \text{eV}
\]

Comparison: Electric vs. Gravitational

\[
\frac{U_{e,p\text{.}\text{elct}}}{U_{e,p\text{.}\text{grav}}} = \frac{1}{4\pi\varepsilon_o} \frac{-e^2}{G \frac{m_e m_p}{r_{1\leftarrow 2}}}
\]

\[
U_{e,p\text{.}\text{elct}} = 9 \times 10^9 \frac{N m^2}{C^2} \left( \frac{1.6 \times 10^{-19} \, C}{6.7 \times 10^{-11} \frac{N m^2}{kg^2}} \right) \left( \frac{9 \times 10^{-31} \, kg}{1.7 \times 10^{-27} \, kg} \right) = 5.6 \times 10^{39}
\]
Return to Rest Energy and Mass

Pair (electron and positron) Annihilation

\[ E = m_e c^2 + m_e c^2 \]

Initial

Electron and positron

Final

Two photons (light pulses)

\[ r_i \approx \infty \]

\[ E = 2m_e c^2 = 2E_\gamma \]

\[ U_{e,p} \left| r_{e \leftarrow p} \right| \left( 0.511\text{MeV} / c^2 \right)c^2 = E_\gamma \]

\[ 0.511\text{MeV} = E_\gamma \]
Return to Rest Energy and Mass
Neutron Decay

\[ E = m_n c^2 \]

\[ E = m_n c^2 = m_e c^2 + m_p c^2 + m_\nu c^2 + K_e + K_p + K_\nu + U_{e,p} + U_{e,\nu} + U_{\nu,\nu} \]

\[ (K_e + K_p + K_\nu) = m_n c^2 - (m_e c^2 + m_p c^2) = 939.6 \text{MeV} - (0.511 \text{MeV} + 938.3 \text{MeV}) = 0.79 \text{MeV} \]

Mass as Energy and Energy as Mass
Box o’ decaying Neutrons

Viewed from outside
Peaking inside

Box’s mass *includes* internal kinetic and potential energies
Return to Rest Energy and Mass

$O_2$ bonding

\[ E_{\text{free}} = 2mc^2 \]
\[ M_{\text{free}} = \frac{E_{\text{free}}}{c^2} = 2m \]

$K + U = 0$

Energy

$r_{\text{free}}$
Return to Rest Energy and Mass

$O_2$ bonding

\[ E_{\text{free}} = 2mc^2 \]
\[ M_{\text{free}} = \frac{E_{\text{free}}}{c^2} = 2m \]
Return to Rest Energy and Mass

$O_2$ bonding

\[ E_{\text{bound}} = 2mc^2 + (K+U) \]
\[ M_{\text{bound}} = E_{\text{bound}}/c^2 = 2m + (K+U)/c^2 \]

Note: would have shed excess energy by emitting photon / light pulse

**Energy / Mass difference**

\[ \Delta M = M_{\text{free}} - M_{\text{bound}} = \left( E_f - E_b \right)/c^2 \]
\[ \Delta M = \left( 2mc^2 - \left( 2mc^2 + (K+U)_{\text{bound}} \right) \right)/c^2 \]
\[ \Delta M = -(K+U)_{\text{bound}}/c^2 = 5\text{eV}/c^2 = 9 \times 10^{-36} \text{kg} \]

Noticeable?

\[ \frac{\Delta M}{2m_o} = 3 \times 10^{-10} \text{ not really} \]
If an iron nucleus were disintegrated, how much K + U energy would be consumed /produced?

**Initial** \[ Fe^{26}_{56} \rightarrow 26p^+ + 30n \]

**Final**

Iron nucleus

Protons and neutrons

\[ E_i = E_f \]

\[ E_{r,Fe} = \sum_{all\,particles} (E_r + K) + \sum_{all\,pairs} U \]

\[ m_{Fe} c^2 = 26 \cdot m_p c^2 + 30 \cdot m_n c^2 + \left( \sum_{all\,particles} K + \sum_{all\,pairs} U \right) \]

\[ m_{Fe} c^2 - \left( 26 \cdot m_p c^2 + 30 \cdot m_n c^2 \right) = \left( \sum_{all\,particles} K + \sum_{all\,pairs} U \right) \]

\[ 52107 MeV - \left( 26 \cdot (939.9 MeV) + 30 \cdot (938.3 MeV) \right) = \left( \sum_{all\,particles} K + \sum_{all\,pairs} U \right) \]

\[ -482 MeV = \left( \sum_{all\,particles} K + \sum_{all\,pairs} U \right) \]

**Noticeable?**

\[ \frac{\Delta mc^2}{m_{Fe} c^2} = 0.009 \approx 1\% \]

Yes
Rest and Electric-Potential and Kinetic

A U-235 nucleus is struck by a slow-moving neutron, so that the merge and become U-236, with mass $M_{\text{U-236}}$. This nucleus is unstable to falling apart – fission. One way it could do so is to first slosh into something of a dumbbell shape, now most of the into two symmetric nuclei, Pd-118, with mass $M_{\text{Pd-118}}$, each has $\frac{1}{2}$ the original number of protons, i.e., $q_{Pd} = 46e$. Having fallen apart, the two palladium nuclei no longer experience a Strong interaction holding them together, just the Electric repulsion of each other’s protons. Subsequently, they accelerate away.

a) What’s the final speed of one of the Pd atoms, when they have sped far, far apart?
b) What is the distance between the Pd atoms just after fission?
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