motion is neither created nor destroyed, but transferred via interactions.

\[ \Delta E = W \]

\[ \gamma_f mc^2 - \gamma_i mc^2 = \sum_{all} \left( \int \vec{F}_{\text{sys}} \cdot d\vec{r}_{\text{sys}} \right) \]

\[ E \equiv \gamma mc^2 \quad E_{\text{rest}} \equiv mc^2 \]

\[ K \equiv E - E_{\text{rest}} = \left( \gamma - 1 \right)mc^2 \approx \frac{1}{2}mv^2 \]
A ball whose mass is 2 kg travels at a velocity of \( <0, -3, 4> \) m/s.

What is the kinetic energy of the ball? 

<table>
<thead>
<tr>
<th>Option</th>
<th>Kinetic Energy (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) (&lt;0, -6, 8&gt;)</td>
<td></td>
</tr>
<tr>
<td>2) (&lt;0, -3, 4&gt;)</td>
<td></td>
</tr>
<tr>
<td>3) 2</td>
<td></td>
</tr>
<tr>
<td>4) 10</td>
<td></td>
</tr>
<tr>
<td>5) 25</td>
<td></td>
</tr>
</tbody>
</table>

\[
K = (\gamma - 1)mc^2
\]

\[
K \approx \frac{1}{2}mv^2
\]
Consider an electron (mass 9e-31 kg) moving with speed \( v = 0.9c \). Its rest energy is 0.81e-13 J, and its (total) particle energy is 1.86e-13 J. What is its kinetic energy?

1) 7.3e-31 J
2) 3.28e-14 J
3) 8.1e-14 J
4) 1.05e-13 J
5) 1.86e-13 J
Rest Energy and Identity

Example: A stationary nucleus whose mass is $3.499612 \times 10^{-25}$ kg undergoes spontaneous “alpha” decay. The original nucleus spits out a He-4 nucleus (2 neutrons & 2 protons) of mass $6.640678 \times 10^{-27}$ kg and the remaining nucleus has a mass of $3.433132 \times 10^{-25}$ kg. When the particles are far apart (so the electric repulsion is negligible) what is the combined kinetic energy?

System: these particles
Active surroundings: none

$\Delta E = W = 0$

$\Delta E_{rest} + \Delta K = 0$

or

$\Delta K = -\Delta E_{rest}$

In the initial mass is the potential for the final motion...

$$K_f - K_i = -(E_{rest,f} - E_{rest,i})$$

$K_f = m_p c^2 - (m_d + m_\alpha) c^2$

$K_f = \left((349.9612 - (343.3132 + 6.640678)) \times 10^{-27} \text{ kg}\right)c^2$

$K_f = \left(7.322 \times 10^{-30} \text{ kg}\right)c^2 = 6.59 \times 10^{-13} J$
Chemistry-Scaled Energy Units

\[
\text{parent nucleus} \rightarrow \text{daughter nucleus} + \alpha \text{ particle}
\]

\[
K_f = \left(7.322 \times 10^{-30} \text{ kg}\right)c^2 = 6.59 \times 10^{-13} \text{ J}
\]

All chemical interactions are fundamentally \textit{electric} interactions, so involve electric force and scale with electron charge.

\[
1 \text{ Joule} = 1 \text{ Joule} \cdot \frac{1 \text{ e}}{1.6 \times 10^{-19} \text{ Coulombs}} = 6.25 \times 10^{18} \frac{\text{ J}}{\text{ C}} = 6.25 \times 10^{18} \text{ eV}
\]

\[
\text{Volt} \equiv \frac{\text{Joule}}{\text{Coulombs}}
\]

Chemical reactions typically involve 10 meV to 10 eV of energy per molecule

\[
K_f = 6.59 \times 10^{-13} \text{ J} \cdot \left( \frac{6.25 \times 10^{18} \text{ eV}}{1 \text{ J}} \right) = 4.12 \times 10^6 \text{ eV} = 4.12 \text{ MeV}
\]

A nuclear reaction involves \textit{millions} of times more energy than does Chemical reaction!
Systematic Approach to Energy Problems

1) Specify a system (the object or object’s you’re interested in)
2) Specify members of the surrounding that interact with the system
3) Specify the initial state
4) Specify the final state
5) Write out the Energy Principle for this system
6) Use the given information to evaluate all terms you can
7) Solve for the target unknown quantity
8) Check units and consistency of sign.
You drop a metal ball from 1 m up. How fast is it going just before it hits the ground?

\[ \Delta E = W \]

\[ W \approx W_{\text{Ball} \leftarrow \text{Earth}} \]

\[ W_{\text{Ball} \leftarrow \text{Earth}} = \int_i^f \vec{F}_{\text{Ball} \leftarrow \text{Earth}} \cdot d\vec{r}_{\text{Ball}} \]

\[ \text{System} = \text{ball} \]

\[ \text{Active members of environment} = \text{Earth} \]

\[ \text{(neglecting air's resistance)} \]

\[ \Delta r_{\text{ball}} = (\Delta y)\hat{y} \]

\[ \Delta y = 1\text{m} \]

\[ \vec{F}_{\text{Ball} \leftarrow \text{Earth}} = mg\hat{y} \]

\[ \text{Not changing} \]

\[ \Delta E_{\text{rest}} + \Delta K = \Delta E \]

\[ K_f - K_i = \Delta E \]

Pretty sure \( v \ll c \), so \( K \approx \frac{1}{2}mv^2 \)

\[ \frac{1}{2}mv_f^2 = \Delta E \approx W_{\text{Ball} \leftarrow \text{Earth}} = mg\Delta y \]

\[ \frac{1}{2}mv_f^2 \approx mg\Delta y \]

\[ v_f \approx \sqrt{2g\Delta y} = \sqrt{2(9.8\text{m/s}^2)(1\text{m})} = 4.4\text{m/s} \]
If we throw the ball down at 5m/s, will have greater, less, or the same speed when it lands?

<table>
<thead>
<tr>
<th>1) greater</th>
<th>2) less</th>
<th>3) The same</th>
</tr>
</thead>
<tbody>
<tr>
<td>greater</td>
<td>less</td>
<td>The same</td>
</tr>
</tbody>
</table>
Slight variation: You *throw* a metal ball straight down at 5 m/s from 1 m up. How fast is it going just before it hits the ground?

\[ \Delta E = W \]

**System = ball**

**Active members of environment = Earth**

(neglecting air’s resistance)

\[ \vec{v}_i = 5 \text{ m/s} \hat{y} \]

\[ \Delta \vec{r}_{ball} = (\Delta y) \hat{y} \]

\[ \Delta y = 1 \text{ m} \]

\[ \vec{F}_{ball \leftarrow \text{Earth}} = mg \hat{y} \]

Not changing

\[ \Delta E_{rest} + \Delta K = \Delta E \]

\[ K_f - K_i = \Delta E \]

Pretty sure \( \nu \ll c \), so

\[ K \approx \frac{1}{2} mv^2 \]

\[ \frac{1}{2} m \left( \nu_{f2}^2 - m \nu_{i2}^2 \right) = \Delta E \approx W_{ball \leftarrow \text{Earth}} = mg \Delta y \]

\[ \frac{1}{2} m \nu_{f2}^2 \approx mg \Delta y + \frac{1}{2} m \nu_i^2 \]

\[ \nu_f \approx \sqrt{2g \Delta y + \nu_i^2} = 6.7 \text{ m/s} \]
If we throw the ball up at 5m/s, will have greater, less, or the same speed when it lands compared with what it had in when thrown 5m/s down?

<table>
<thead>
<tr>
<th>1) greater</th>
<th>2) less</th>
<th>3) The same</th>
</tr>
</thead>
</table>

Another Slight variation: You throw a metal ball straight up at 5 m/s from 1 m up. How fast is it going just before it hits the ground? What changes now?

\[ \Delta E = W \]

\[ \vec{v}_i = -5 \, m/s \, \hat{y} \]

System = ball

Active members of environment = Earth (neglecting air’s resistance)

\[ \Delta \vec{r}_{ball} = (\Delta y) \hat{y} \]

\[ \Delta y = 1 \text{m} \]

\[ \vec{F}_{ball \leftarrow \text{Earth}} = mg \hat{y} \]

Not changing

\[ \Delta E_{rest} + \Delta K = \Delta E \]

\[ K_f - K_i = \Delta E \]

Pretty sure \( v << c \), so \( K \approx \frac{1}{2} m \nu^2 \)

\[ \frac{1}{2} m (\nu_f^2 - \nu_i^2) = \Delta E \approx W_{Ball \leftarrow \text{Earth}} = mg \Delta y \]

\[ \frac{1}{2} m \nu_f^2 \approx mg \Delta y + \frac{1}{2} m \nu_i^2 \]

\[ \nu_f \approx \sqrt{2g \Delta y + \nu_i^2} = 6.7 \, m/s \]
**Work – Energy Relation**

**Example:** An electron (mass $9 \times 10^{-31}$ kg) is traveling at a speed of $0.95c$ in an electron accelerator. Over what distance would an electric force of $1.6 \times 10^{-13}$ N have to be applied to accelerate it to $0.99c$?

$$\vec{v}_i = (0.95c)\hat{y} \quad \text{System= electron}$$

Active members of environment = Accelerator

$$\Delta E = W_f$$

$$\left( \gamma_f mc^2 - \gamma_i mc^2 \right) = \int F \cdot d\vec{r}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \left( \frac{v}{c} \right)^2}}$$

$$\gamma_f - \gamma_i \cdot mc^2 = F \Delta y$$

$$\left( \gamma_f - \gamma_i \right) \frac{mc^2}{F} = \Delta y$$

$$\Delta r_e = (?)\hat{y}$$

$$F_{e\rightarrow \text{accelerator}} = \left(1.6 \times 10^{-13} \text{ N}\right)\hat{y}$$

$$\Delta y = \left( \frac{1}{\sqrt{1 - (0.99)^2}} - \frac{1}{\sqrt{1 - (0.95)^2}} \right) \left( 9 \times 10^{-31} \text{ kg} \right) \left( 3 \times 10^8 \text{ m/s} \right)^2 \frac{1}{1.6 \times 10^{-13} \text{ N}}$$

$$\Delta y = 37 \text{ m}$$

$$\vec{v}_f = (0.99c)\hat{y}$$
Let's say the Wizard of Oz's balloon has a total mass of 1000 kg (including the Wizard in the basket) and has most of its volume in the balloon which has a radius of 10 m. If the Scarecrow, whose mass is 50 kg (he is only made of straw after all) grabs a hold, when it starts lifting off, how fast will it be going when it gets 3 m up, at which point the Scarecrow lets go and drops? Recall that air has a density of \(1.3 \times 10^{-3} \text{ g/cm}^3\).
Let’s say the Wizard of Oz’s balloon has a total mass of 1000 kg (including the Wizard in the basket) and has most of its volume in the balloon which has a radius of 10 m. If the Scarecrow, whose mass is 50 kg (he is only made of straw after all) grabs a hold, when it starts lifting off, how fast will it be going when it gets 3 m up at which point the Scarecrow lets go and drops? Recall that air has a density of $1.3 \times 10^{-3} \text{ g / cm}^3$.

Now you try. How much work did the balloon do on the scarecrow during his brief ride?

System
Active Surroundings
Initial State
Final State
Work-Energy Relation
Algebra
Numbers
Check units and sign
Work by Changing Force

\[ W = \sum_{i=1}^{20} \vec{F}_i \cdot \Delta \vec{r}_i \]

\[ W = \lim_{\Delta \vec{r} \to 0} \sum_{\text{initial}}^{\text{final}} \vec{F} \cdot \Delta \vec{r} = \int_{\text{init}}^{\text{fin}} \vec{F} \cdot d\vec{r} \]
Work by Changing Force

Spring

\[ W = \lim_{\Delta r \to 0} \sum_{\text{final}}^{\text{initial}} \vec{F} \cdot \Delta \vec{r} = \int_{\text{init}}^{\text{fin}} \vec{F} \cdot d\vec{r} \]

\[ F = -k_s x \]

\[ W = \text{Difference between triangles' areas} \]

\[ W = \int_{i}^{f} -k_s x \cdot dx \]

\[ W = -\frac{1}{2} k_s x^2 \bigg|_{i}^{f} \]

\[ W = -\frac{1}{2} k_s \left( x_f^2 - x_i^2 \right) \]
In an up-coming lab, you’ll use a spring-loaded gun to fire a metal ball. If we know the ball’s mass, the stiffness of the spring and how much it’s been stretched, then we should be able to predict where the ball will hit the ground. Say the spring is initially compressed by 0.04m, and when it launches the ball it stretches to a compression of 0.01m; if the ball is 0.05kg and the gun is mounted 0.8m above the ground, then how far over will the ball hit the ground?
motion is neither created nor destroyed, but transferred via interactions.

\[ \Delta E = W \]

\[ \gamma_f mc^2 - \gamma_i mc^2 = \sum_{all} \left( \int \vec{F}_{\rightarrow sys} \cdot d\vec{r}_{sys} \right) \]

\[ E = \gamma mc^2 \]

\[ E_{rest} = mc^2 \]

\[ K = E - E_{rest} = (\gamma - 1)mc^2 \approx \frac{1}{2}mv^2 \]

Fri. 6.5-.7 (.22) Rest Mass, Work by Changing Forces Columbia Rep 3pm, here

RE 6.b (last day to drop)

Wed. 6.8-.9 (.18, .19) Introducing Potential Energy...

RE 6.c

Tues. ...

HW6: Ch 6 Pr’s 58, 59, 99(a-c), 105(a-c)