Physics 231 – Lab 11
Angular Momentum

(equipment: angular momentum apparatus – base, bar, sliding masses, 2 photogates, catcher, launcher, meter stick, scale)

Objectives

In this lab, you will do the following:

- Determine the moment of inertia for the apparatus.
- Get practice applying the Angular Momentum Principle to different systems.

Background

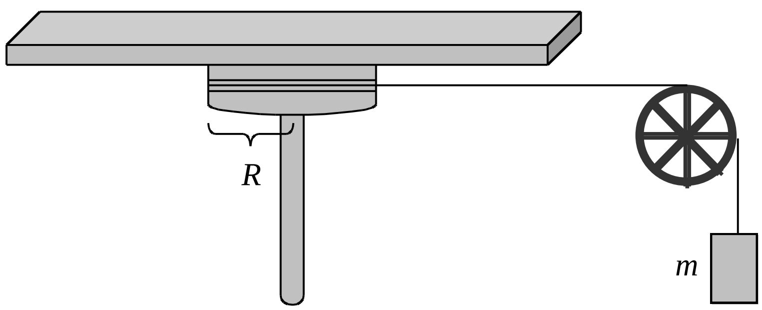
According to the Angular Momentum Principle, the change in total angular momentum about point A in a short time Δt is $\Delta\vec{L}_{tot,A} = \vec{\tau}_{net.on.A}\Delta t$, where $\vec{\tau}_{net.on.A}$ is the net, external torque about the point. For a point particle, the angular momentum relative to some chosen reference point is $\vec{L}_A = \vec{r} \times \vec{p}$, where $\vec{r}$ is the particle's position and $\vec{p}$ is its momentum (both relative to the reference point.) For a solid object, the rotational angular momentum about some axis is $\vec{L}_{rot} = I\vec{\omega}$, where I is the moment of inertia (relative to the axis) and $\vec{\omega}$ is the angular velocity. The kinetic energy associated with the rotation of a solid object is $K_{rot} = \frac{1}{2}I\omega^2$.

I. Moment of Inertia of the Apparatus

You'll start by developing a mathematical model of the system we'll study, then you'll experiment with the system.

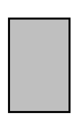
A. Developing a Model

You'll develop this model to help you express the object's moment of inertia in terms of its motion, which you will later observe. A simple diagram of the experimental setup is shown below.



First, consider just the hanging mass as a system.

- On a white board, make a diagram showing the forces on the mass while it is falling and pulling a string. Label the forces. Note: be careful about their relative magnitudes.



- Apply the Momentum Principle to relate the applied forces to the rate of change in the speed ($\Delta v_m/\Delta t$). Use *downward* as the positive direction. Solve your equation for the force the string is exerting.

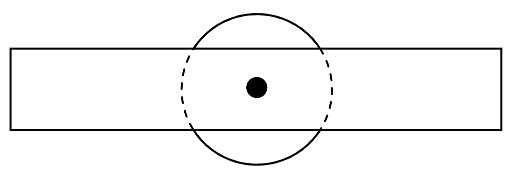
Second, relate the linear motion of the mass to the rotational motion of the beam.

Consider: How is the speed of the falling mass related to the speed of the points on the outer *edge* of the disc where the string is wound around it?

- Determine an expression relating the speed of the mass, v_m, to the speed of the edge of the ring, v_R.
- What's the relation between the speed at the edge of the ring, v_R, and the ring's angular speed, ω?
- So, by connecting these two expressions, you have an expression relating the speed of the mass, v_m, to the angular speed of the apparatus, ω.
- Finally, relate the change in the speed (Δv_m) to the change in the angular speed ($\Delta \omega$).

Third, consider the rotating beam (and the wheel mounted beneath it) as a system (this does not include the falling mass.)

- On a whiteboard, make a sketch showing the apparatus from above (see below) and the forces on the beam & cylinder that are responsible for torques about its axis (i.e., you can ignore forces directly applied to the axis.) Be sure that forces are drawn where they are applied. Label the radius of the cylinder that the string is wrapped around as *R*. Assume that the string will be wrapped counterclockwise.



- Apply the Angular Momentum Principle to relate the rate of change in the angular speed ($\Delta\omega/\Delta t$) to the applied forces. Label the moment of inertia as *I*. Ignore friction. Solve the equation for the moment of inertia, *I*.

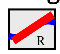
Finally, combine equations 1, 2, & 3.

- Solve for the moment of inertia in terms of the rate of change of the angular speed ($\Delta\omega/\Delta t$), which you will measure in the following experiment. That is, substitute equation 2 into equation 1 as to eliminate v_m, and then substitute the resulting equation into equation 3.

B. Experiment

Now you're ready to experiment with the real thing, observe its motion, and thus experimentally determine its moment of inertia.

- Open the file "AngularSpeed.cmbl" which will measure the angular speed by determining how quickly the spokes of the wheel pass the photogate. Plug the photogate into the "DIG/SONIC 1" port.
- Determine the radius of the largest ring on the apparatus where the string will be wrapped and record it below. It may be easiest to first determine its circumference by measuring the length of string that wraps around it once.

- Wrap the string three times around the largest ring on the apparatus. Only make a single layer of string. If there is a second layer, it pulls from a different radius than what you measured.
- While holding the apparatus so that it can't rotate, hang a 500-g mass from the end of the string.
- Press "Collect," wait a few seconds, and let the apparatus spin. Be careful because it will get pretty fast.
- Select a straight section of the graph of the angular speed versus time while the mass is falling. Press the linear fit button  to find the slope, which is the rate of change of the angular speed. Record that measurement below.
- Calculate the moment of inertia for the apparatus using the expression based on the model in Part A. Show all of your work.

C. Comparison with the Theoretical Value

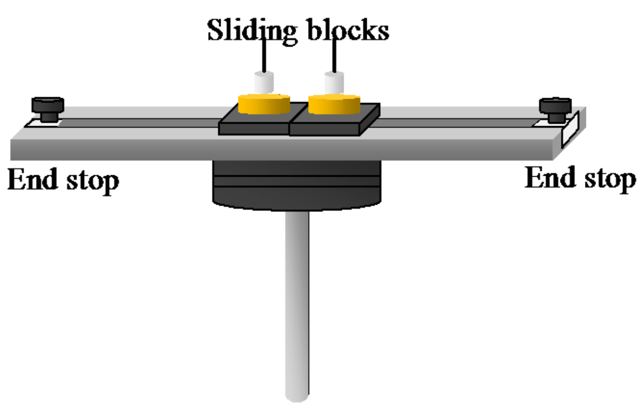
Now that you've performed and interpreted the experiment, let's compare with the theory. Given the object's mass distribution, we can calculate what the moment of inertia should be. The moment of inertia for a thin, uniform beam that is rotated about its center is $I = \frac{1}{12} M \ell^2$, where M is the mass and ℓ is the length.

- Remove the beam of the apparatus and measure its mass and length.
- Calculate the theoretically expected value of the moment of inertia. Show your work.

If this theoretically-determined value isn't close enough to the experimentally-determined one, you should double check your measurements and calculations.

II. A System with a Changing Moment of Inertia

You will perform an experiment in which the moment of inertia of the system will change, which will result in a change in the angular speed. For this part, take the rotational apparatus and the sliding blocks as the system (see illustration below.) You'll start with the two sliding blocks near the center of the beam, as illustrated. They'll be joined by a piece of thread. Once the system is spinning, you'll burn the thread, and the blocks will slide out until they hit the end stops. You'll analyze angular momentum and kinetic energy through this transformation.



Pause and Consider. If the system has negligible external torque applied to it, how should the angular momentum compare before and after this change?

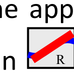
A. Initial Moment of Inertia

- Measure the mass of one of the sliding blocks (with attached weights).
- Measure the mass of one of the end stops (a screw and a bolt).
- Measure the mass of one of the stops (a screw and a nut) that will attach to track on the apparatus.
- Slide the blocks onto the track so the white marks for measuring their positions are next to the scale. Slide the end stops on and tightly secure them at the ends of the track so they and the blocks won't fly off!
- Record the distance from the center of one end stop to the center of the track.
- To secure the two blocks to each other, loosen the nuts that hold down the brass weights and wrap a strand of thread beneath them, from one block to the other, then tighten the nuts back down on thread. This should hold the blocks snugly against each other. Center the pair of blocks on the center of the track and measure the initial distance of one (its white mark) from the center of the track.
- Use a white board to derive, in terms of m_s, r_s, m_b, r_b, M and l, a symbolic expression for the whole system's (two blocks, two fixed stops, and one long beam) moment of inertia.
- Based on your measurements, what's the initial value of the moment of inertia?

B. Angular Speeds

- Close LoggerPro. Open the file "Angle.cmbl" which will measure the angle that the apparatus has turned as a function of time.

Read through all of the following instructions before performing the experiment.

- Hold the base of the apparatus and spin the beam *quickly*.
- Press "Collect" in the LoggerPro program.
- Use the lighter to burn the thread connecting the sliding blocks.
- Determine the initial (just before the thread burns) and final (just after the blocks hit the end stops) angular speeds of the apparatus by selecting the appropriate straight portions of the angle versus time graph and pressing the linear fit button .

C. Final Moment of Inertia

- Measure the final distance of a block from the center of the track.
- Calculate the final moment of inertia of the whole system, based on the equation you derived and your measurements.

D. Angular Momentum

- Given your initial angular speed and moment of inertia, calculate the magnitude of the initial angular momentum of the system.

- Calculate the magnitude of the final angular momentum of the system.

- With negligible external torques during the 'explosion', the system's angular momentum should be conserved. Check that: Calculate the percent difference to determine whether or not angular momentum was conserved during this transformation (given our experimental uncertainties, a difference less than 10% is consistent with angular momentum being conserved) .

E. Energy

- Calculate the initial rotational kinetic energy of the system.
- Calculate the final rotational kinetic energy of the system.
- Calculate the change in the rotational kinetic energy of the system.

Pause and Consder: What other form(s) of the system's energy change during the transformation to account for the lost rotational kinetic energy?

III. Firing a Ball at the Apparatus

You will shoot a ball into a catcher mounted at the end of the beam and consider angular momentum in the system of ball, beam, and catcher.

- Find the mass of the ball and of the catcher (including the bolt that holds it in place).
- Remove the end stops and sliding blocks from the beam.
- Attach the catcher near one end of the rotational apparatus. The catcher should be oriented so that the ball will enter perpendicular to the beam. Record the distance of the catcher's *center* from the middle of the apparatus.
- Calculate what the *final* moment of inertia of the system will be *after* the ball is shot into the catcher.
- Line up the launcher to shoot the ball into the catcher, making sure the beam will not hit the launcher when it rotates. Test that it works.
- Push the ball into the launcher so that it clicks three times. Press the "Collect" button and wait a few seconds, then shoot the ball into the catcher.
- Use the graph to determine the final angular speed (just after the collision).

To test how well the conservation of angular momentum applies, you can calculate what the ball's launch speed should have been (based on the subsequent motion of the system) and what it was (based on direct measurement.)

- On a whiteboard, use conservation of angular momentum to determine an expression for what the ball's launch speed must have been in terms of I_f, ω_f, r_c, and m_b.
- Using the values you've measured and calculated, calculate the ball's speed just before the collision based on your measurements.
- To more directly measure the ball's launch speed, make sure the bracket holding two photogates is attache to the launcher. Unplug the photogate that's watching the beam's rotation and plug in the launcher's two photogates - the photogate closest to the launcher plugs into "DIG/SONIC 1" and the other plugs into "DIG/SONIC 2". Close LoggerPro and reopen it through the file "Launch Timer".
- Measure the distance between the middles of the photogates.
- Push the ball into the launcher so that it clicks three times. Press the "Start" button and shoot the ball into the box. Record the time it takes for the ball to pass between the photogates.
- Use the last two measurements to calculate the speed of the ball.

Question: How does the speed determined from the collision compare with the more direct measurement of the speed using the photogates? Calculate the percent difference. If much greater than 10%, find and fix your mistake.