Today:	**10** Simple Harmonic Motion & Elasticity 1st ½		
Monday:	**10** Simple Harmonic Motion & Elasticity 2nd ½		HW1: 2,4,8
Wednesday:	**16** Waves & Sound 1st 1/3		HW2:
Friday:	**16** Waves & Sound 2nd 1/3	HW1 redo,	HW3:
Lab:	**Lab 1**: Harmonic Motion		

Materials

Demos:

Mass on a spring hung from a beam (spring, mass, beam)

Tuning Fork & resonance box (fork, box, microphone, oscilloscope)

Guitar string (string, mass, mass hook, wires, oscilloscope)

Spring Scale (spring scale, something to hang it from, mass)

Handouts:

- Exams & old homework
- Syllabus
- Lab 1
- Office Hour Survey

Brief Intro.

- **Instructor: Eric Hill**
  - **Edu.:** B.A. Carleton College, PhD U. of Minn. – Condensed Matter Physics
  - **Research:** STM studies of surface processes of individual molecules.
  - **Schedule:** 221 and 107 this semester (so I'll be a little less available than I was last).
  - **Office Hours:** To Be Determined
- **Physics 221**
  - 2nd semester of a 2 semester algebra-based intro. Physics sequence. We hit the ground running this semester – assuming that your physics is up to speed thanks to Phys. 220 or a similar course at another college or in High School.

Administrative:

- **Take Role**
  - Check who's there & that they are in the right lab section
  - The following folks may wish to see me after class
    - Brandon doesn't have a lab section
    - Karen isn't registered yet
    - About 7 students don't appear to have taken ~prerequ.
      - Likely due to 220 not having been listed as a pre-requisite
      - Strongly encourage to consider dropping 221 and taking either 231 (first in the Calc. Based sequence) or 220 next Fall and 221 next Spring.
- **Office Hour Survey**
  - **Instructions:** Grayed out times are off limits, mark an "N" in the time blocks where you have a conflict, mark a "Y" in the blocks that are good for you, pass it on.
  - Open door policy, whether it's an office hour or not, I'm generally available in the white time blocks.
- **Syllabus**
  - **What's changed**

        - **No Group Problems –** you've learned the basics of problems solving technique, now it's up to you
        - **No Pre-Lecture Questions** – you've seen for yourself the value of reading before lecture
        - **No Pre-Lab Questions** – you've seen for yourself the value of thinking about lab before performing lab
        - **Homework**
            - Assigned daily, over material just covered – tighten the loop
            - Graded Boolean, but with the ability to redo – improve homework skills & decrease grading time
            - Read Homework section of Syllabus & see schedule.

- **Course Background**
    - **Physics Program**:
        - **Experiment**: Observing and the world around us & translating those observations into the vocabulary of mathematics (numeric quantities).
        - **Theory**: Using the grammar of mathematics (equations) to describe or model motion and interaction in the real world – start with info. about initial conditions & propagate that info through mathematical tools to predict info about final conditions.
        - **Refinement**: a feedback loop of experiment, comparison with theory, and modification of experiment and theory until they agree.
    - **Physics 220**
        - **Classical Mechanics:** The study of Motion and the transfer of Motion via Interactions.
        - **Fundamental Principle**: Motion is neither created nor destroyed, but transferred via interactions.
        - **Topics**
            - What topics did we cover last semester?
            - **Kinematics** (motion) in 1 & 2 –D and rotational
                - Kinematics tools
                    - Time
                    - Distance
                    - Position
                    - Displacement
                    - Speed
                    - Velocity
                    - Acceleration
                    - Angular versions of the above
            - **Dynamics** (interactions) in 1 & 2-D and rotation
                - Dynamics tools
                    - Force (Newton's 3 laws)
                    - Momentum & Impulse
                    - Work & Energy (Kinetic & Potential)
                    - Torque & Angular Momentum
            - **Systems of Particles**
                - Rotation of Extended bodies
                - Fluids

                - Heat
                - Thermodynamics
                    - Ideal Gas Law & Kinetic Theory
            - **Skills**
                - Mathematical
                - Problem Solving
                - Quantitative Reasoning
                - Study Skills
                - Homework
                - Teacher Comprehension
        - **Carry-over from Phys 220**.
            - The skills honed last semester will be called upon this semester. Many of the topics of last semester will be built upon, in part or in whole, this semester.
        - **What's new**
            - Topics fall under three main, overlapping, headings
                - **Harmonic Motion:** A kind of motion
                    - Wave phenomena:
                        - Sound (systems of particles)
                        - Light (Electricity and Magnetism)
                            - Optics
                - **Electricity & Magnetism**: A kind of interaction
                    - E & M forces and fields
                    - Circuitry
                    - Light & Optics
                - **Modern Physics** (refinements of mechanics)
                    - Relativity (high speed mechanics)
                    - Quantum (small mechanics)
                        - Atoms
                        - Nuclei
                        - Radiation

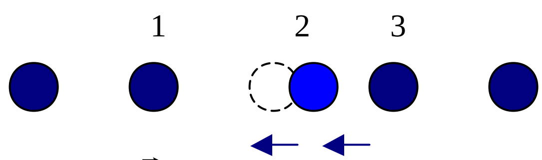
Without Further Ado

10 Simple Harmonic Motion and Elasticity

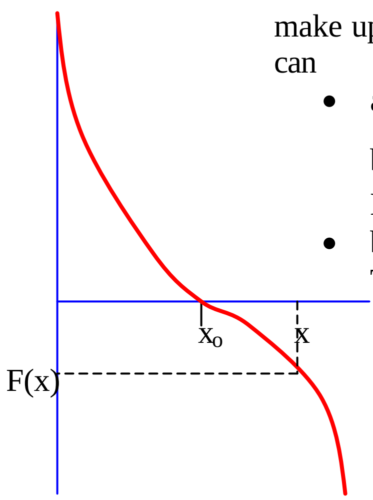
- **Intro:** We kick this off by talking about Hooke's law, $\vec{F}_{Sp\rightarrow} = -k_{sp}\Delta\vec{x}$. It says that the force with which a spring pulls back to its equilibrium length, F_{sp}, is directly proportional to how far it's been stretched or compressed from that length, Δx.
- I should point out immediately that this does not describe an interaction on the fundamental level, in terms of an electric, magnetic, weak, strong, or gravitational forces. That fact begs the question 'how *does* it relate to the fundamental forces?'
- Indeed, I want to understand where this simple, yet mystical, relationship comes from. To see that, we're going to build a spring, atom by atom. Not only will we get a sense of where this comes from, but we'll also see that this relationship scales straight down to the individual atomic level! And if that beauty isn't enough to justify our digging so deeply, how about this: if this holds for individual atoms, then it holds for most anything made of individual atoms, i.e., not just springs, but everything around you! Ex. Atoms in their

molecules, hairs in your ears, heads on drums, and even (disastrously) suspension bridges.

- With that established, we'll look at what kind of motion a force like this causes, Simple Harmonic Motion. Understanding atomic scale nature of Hooke's law, you'll appreciate that it is no mere coincidence that so much solid mater executes (approximately) Simple Harmonic Motion.
-
- **Inter-Atomic Restoring Force**
  - Consider a chunk of material, say, a rod of iron. Imagine zooming in so you can see individual atoms. From our discussion of thermodynamics, we know that the atoms in the rod have some kinetic energy, they're moving a little. But we also know from the simple fact that the rod doesn't desintegrate, that there are some bonds that oppose this motion and keep the atoms relatively stationary. When an atom gets thermally knocked a bit to the left, the bonds it has with its neighbors must conspire to send it back home again. The net force due to these bonds is of a class called a "restoring" force, for it *restores* the atom back to its original position.



$$\vec{F}_{net\to 2} = \vec{F}_{1\to 2} + \vec{F}_{3\to 2}$$



  - To speak *exactly* quantitatively about this restoring force, we'd need to know the number and distribution of all the charged particles that make up the individual atoms. But to speak only *approximately* we can
    - a) use that we know that the net force must depend on the atom being displaced, i.e., $\vec{F}(x)_{net\to 2}$, otherwise all mater above 0 K would vaporize
    - b) use that most mathematical functions can be expanded in a Taylor series.
      - Taylor series of some function which depends on x, F(x), expanded about an evaluation point of x_o:

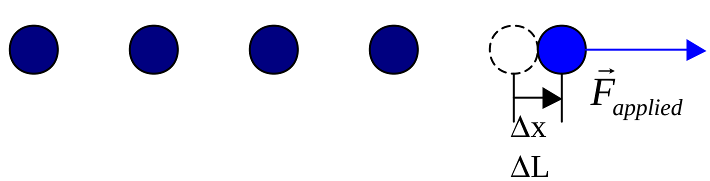
$$F(x) = F(x_o) + \left.\frac{dF(x)}{dx}\right|_{x_o}(x - x_o) + \frac{1}{2}\left.\frac{d^2F(x)}{dx^2}\right|_{x_o}(x - x_o)^2 + \ldots$$

    - So, we can represent our net force as an expansion series. We are imagining that the atom only strays a little from its equilibrium position, x_{eq}, so that will be the point about which we expand the function.

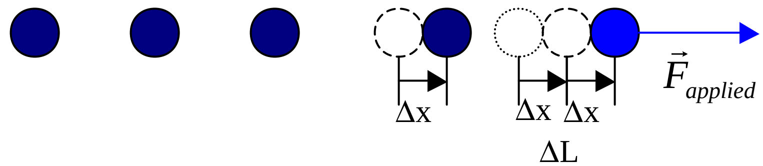
$$F(x)_{net} = \underbrace{F(x_{eq})_{net}}_{0} + \left.\frac{dF(x)_{net}}{dx}\right|_{x_{eq}} \left(x - x_{eq}\right) + \underbrace{\frac{1}{2}\left.\frac{d^2F(x)_{net}}{dx^2}\right|_{x_{eq}} \left(x - x_{eq}\right)^2 + \ldots}_{0}$$

- The first term in the series, net force evaluated at the equilibrium point, is by definition 0. That makes the second term the first remaining. We're talking about small displacements, the second term is proportional to something small, the third term is proportional to something small squared, or qualitatively speaking, something *really* small, subsequent terms are even smaller still. So, as long as we are talking about only *small* displacements, the net force can be approximated as $F(x)_{net} \approx \left.\frac{dF(x)_{net}}{dx}\right|_{x_{eq}} \left(x - x_{eq}\right)$
- The exact value of the derivative of the net force, evaluated at the equilibrium point, depends on all the nitty-gritty info of charge distribution etc. but at the end of the day, it is just some number, some constant, furthermore, it must be negative since that would give the observed behavior of the atom being pulled *back* to its equilibrium point. So we can replace the notation for the derivative… with the notation for just some negative number: $\left.\frac{dF(x)_{net}}{dx}\right|_{x_{eq}} \equiv -k_{atom}$.
- $F(x)_{net} \approx -k_{atom}\left(x - x_{eq}\right)$
  $F(x)_{net} \approx -k_{atom}\Delta x$
- This result looks a lot like Equation 10.2, Hooke's Law. The only difference is that Hook's law is talking about a spring, not a single atom, but the form of the equation is the exact same. We'll see why.

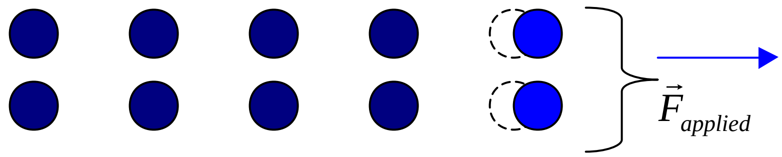
- **Scale up**
  - **Series**
  - One way of looking at this is to say that if I were able to reach into the chain of atoms and pull just one atom to the right with force F, it would move a distance Δx., or the chain would lengthen by ΔL = Δx.. $F_{applied} = k_{atom}\Delta x = k_{atom}\Delta L$



  - Now say it and its neighbor along the chain are both free to move, it takes just a little showing, but for the same force, I get the same amount of stretch out of both of them, or twice the total stretch. ΔL = 2Δx. $F_{applied} = k_{atom}\Delta x = k_{atom}\dfrac{\Delta L}{2}$



- If three atoms were free to move, I'd get three times the displacement…if N atoms were free to move, I'd get N times the displacement. $F_{applied} = k_{atom}\Delta x = k_{atom}\dfrac{1}{N_{Long}}\Delta L$.
- **Parallel**



- Now say don't just have one chain of atoms to stretch, but a rod of atoms, say 10 atoms wide and 20 atoms tall. Now to the same applied force is distributed among all 10 × 20 = 200 atoms. Each only gets 1/200th of the force, and so moves 1/200th of the distance. The resulting equation looks like
- $F_{applied} = k_{atom}\dfrac{N_{Wide}N_{deep}}{N_{Long}}\Delta L$
- Note: This relationship holds for *any* uniform bulk solid. The basic idea can be extended to a metal bar or to your desk top – This describes how hard it is to squash or stretch something.

10.1 The Ideal Spring and Simple Harmonic Motion

- A spring is just such a collection of atoms, so many atoms long, so many wide, and so many deep.

10.1.1 Equation 10.1

- A spring's compression or expansion is proportional (approximately) to the force pushing or pulling on it.
- $F_{\rightarrow Sp} = k_{sp}\Delta L$
    - $F_{\rightarrow Sp}$ is the force applied to the spring to stretch or compress it.
    - ΔL is the change in its length due to stretching or compressing
    - k_{sp} is the called the spring constant, has units of N/m. It determines a spring's "stiffness" – the bigger k, the stiffer the spring, i.e., the harder to stretch or compress. The spring constant relates to fundamental properties, the number of atoms across the cross-section of the spring, $N_w \times N_d$ the number of atoms long the spring is, N_L and the

fundamental inter-atomic forces by $k_{sp} = k_{atom}\frac{N_W \times N_d}{N_L}$, and

$$k_{atom} = -\left.\frac{dF(x)_{net}}{dx}\right|_{x_{eq}}$$

10.1.2 Example Problem: Similar Springs, different lengths

Demo: Similar springs of different length.

Hang a mass from ½ along a spring, measure how far it stretches. Ask if I hang the same mass from the end of the spring, will it stretch less, just as much, or more?

It stretches more, twice as much to be exact. Why? Look at our relationship between the length of a chain of atom and the spring constant:

$k_{sp} = k_{atom}\frac{N_W \times N_d}{N_L}$. Twice as many atoms long, twice as much stretch.

- Say you hang a mass from a spring and it stretches by 0.3 m. Then you cut the spring in half and hang the same mass from just one of the halves. By how much will the spring be stretched?
  - o $\Delta L_1 = 0.2$ m, $\Delta L_{1/2} = ?$
  - o **Equations**
    - $F_{\to Sp} = k_{sp.1}\Delta L_1$, $F_{\to Sp} = k_{sp.1/2}\Delta L_{1/2}$,
    - $k_{sp.1} = k_{atom}\frac{N_W \times N_d}{N_{L.1}}$, $k_{sp.1/2} = k_{atom}\frac{N_W \times N_d}{N_{L.1/2}}$
    - $N_{L½} = ½ N_{L1}$
  - o **Algebra**
    - $\Delta L_{1/2} = \frac{F_{\to Sp}}{k_{sp.1/2}}$

      Find $k_{sp\ ½}$
      - $k_{sp.1/2} = k_{atom}\frac{N_W \times N_d}{N_{L.1/2}}$

        Find $N_{L½}$
        - o $N_{L½} = ½ N_{L1}$
      - $k_{sp.1/2} = k_{atom}\frac{N_W \times N_d}{\frac{1}{2}N_{L.1}} = 2k_{atom}\frac{N_W \times N_d}{N_{L.1}}$

        Find $k_{sp\ 1}$
        - o $k_{sp.1} = k_{atom}\frac{N_W \times N_d}{N_{L.1}}$
      - $k_{sp.1/2} = 2k_{sp.1}$

        Find $k_{sp\ 1}$
        - o $k_{sp.1} = \frac{F_{\to Sp}}{\Delta L_1}$
      - $k_{sp.1/2} = 2\frac{F_{\to Sp}}{\Delta L_1}$
    - $\Delta L_{1/2} = \frac{F_{\to Sp}}{2\frac{F_{\to Sp}}{\Delta L_1}} = \frac{1}{2}\Delta L_1$

- **Numbers**

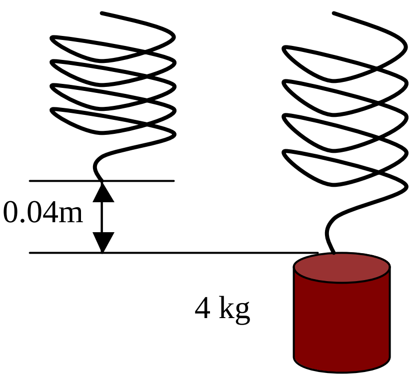
$$\Delta L_{1/2} = \frac{1}{2}\Delta L_1 = \frac{1}{2}0.3m = 0.15m$$

10.1.3 Example Problem: Spring Scale

- Just as a mercury thermometer makes use of the linear relationship between the mercury column's Temperature and its Length, A spring scale makes use of the linear relationship between the Force tugging on (or compressing) a spring and its Length.

Demo: Spring Scale

- Say the spring of the scale lengthens by 0.04 m when reports holding a 4 kg weight. What is the Spring constant, k_{sp}?



- $\Delta L = 0.04$ m
- m = 4 kg
- $F_{\to Sp} = k_{sp}\Delta L$
  $F_{\to Sp} = w = mg$
- $k_{sp} = \dfrac{F_{\to Sp}}{\Delta L}$
  - $F_{\to Sp} = mg$
- $k_{sp} = \dfrac{mg}{\Delta L}$
- $k_{sp} = \dfrac{4kg \cdot 9.8m/s^2}{0.04m} = 980\ {}^{N}\!/\!{}_{m}$

10.1.4 Hooke's Law

- We have an equation for the force applied *to* a spring (and the resulting lengthening or compressing); by Newton's third Law, we can say that the force applied *by* the spring is equal and opposite to this: $F_{Sp\to} = -F_{\to Sp} = -k_{sp}\Delta L$
- **Equation 10.2** $\vec{F}_{Sp\to} = -k_{sp}\Delta\vec{x}$
- **Note:** This is not a fundamental force (gravitation, electric, magnetic,…), rather it is a mathematical result of the summing approximations of many individual, fundamental interactions. In most cases, the underlying forces are electric, but for practical purposes, we needn't model on that level.

10.2 Simple Harmonic Motion and the Reference Circle

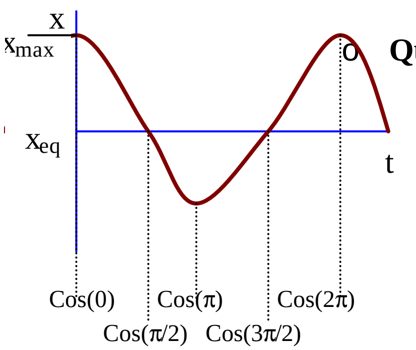
- **Intro.** So far, we've only talked about static situations – the spring is stretched to some new length, and there it stays. Now we'll think about motion. Say you have a mass on a spring's end, and you pull it out of equilibrium. What happens?

Demo: mass on spring, let it bob

- The mass bobs up and down. Let's think about what's happening at the two extremes and at the equilibrium point in the middle.
  - At the bottom, the mass is stationary, p = 0, but there is a net force, $\vec{F}_{met} = -k_{sp}\Delta\hat{y} - m\vec{g} = (k_{sp}\,|\Delta y| - mg)\hat{y} = \dfrac{\Delta\vec{p}}{\Delta t}$**,** pulling up, so momentum changes from 0 to up – the mass moves up,.
    - As the mass rises toward the equilibrium position, the net force continues to accelerate the mass faster and faster,

meanwhile the net force itself is shrinking as the spring becomes less stretched. $\vec{F}_{Sp\rightarrow} = -k_{sp}\Delta\vec{x}$

- In the middle, the spring is in equilibrium, there are no forces, so the momentum remains unchanged. Since the momentum was pointing up before the mass arrived in equilibrium, it continues pointing up, and the mass continues to move up. $0 = \dfrac{\Delta\vec{p}}{\Delta t}$
    - As the mass rises above equilibrium, the spring compresses and produces a force pushing down toward the equilibrium position. This is the opposite direction of the mass's momentum, so the momentum changes to decrease. $\vec{F}_{met} = -k_{sp}\Delta\hat{y} - m\vec{g} = (-k_{sp}\,|\Delta y| - mg)\,\hat{y} = -\left|\dfrac{\Delta p}{\Delta t}\right|\hat{y}$
- At the top, the mass has lost all of its momentum and comes to a halt. At this point the spring has maximum stretch & maximum force, so it pulls the mass back down.



- **Qualitative Plot of Position Vs. Time**
    - Start's at maximum position, $X_{max,}$ initially at rest
    - It begins moving toward the equilibrium position, slowly at first
    - Under the constant application of the spring force, it moves faster and faster
    - By the time it arrives at the equilibrium position, it is moving its fastest, but also at that time the spring force has dwindled to nothing.
    - The mass below equilibrium now feels a slight pull back up, so it begins to decelerate
    - It continues to loose speed until it bottoms out at $-X_{max}$.
    - Then the process replays in reverse – pulling the mass back up…
    - **Q:** What mathematical (trig) function does this plot look like?
        - Cosine
    - **Q:** What is the argument of Cosine when it is maximized, crosses zero, minimized, crosses zero again?
- **Mathematical description of Motion**
    - **Displacement**
    - To make things simplest, we'll just consider a mass on a spring lying horizontally on a frictionless table, i.e., a mass subject only to the force of the spring. Furthermore, we'll set the origin at the equilibrium position.
    - $F_{Sp\rightarrow m} = -k_{sp}\Delta x = -k_{sp}x$,
    - $F_{net} = ma = m\dfrac{dv}{dt} = m\dfrac{d\frac{dx}{dt}}{dt} = m\dfrac{d^2x}{dt^2}$
    - $F_{net} = F_{Sp\rightarrow m}$
    - $$m\frac{d^2}{dt^2}x = -k_{sp}x$$
      $$\frac{d^2}{dt^2}x = -\left(\frac{k_{sp}}{m}\right)x$$

- Any one know a mathematical function whose second derivative is the negative of the function?
    - What is the derivative (slope) of Cosine?
        - – Sine
    - What is the derivative (slope) of Sine?
        - Cosine
- $x = x_{\max} \cos(wt)$

- Plug this in and see what we get.

$$\frac{d^2}{dt^2}\left(x_{\max}\cos(wt)\right) = -\left(k_{sp}/m\right)x_{\max}\cos(wt)$$

$$x_{\max}\frac{d^2}{dt^2}\left(\cos(wt)\right)$$

$$x_{\max}\frac{d}{dt}\left(\frac{d}{dt}\cos(wt)\right)$$

$$x_{\max}\frac{d}{dt}\left(-w\sin(wt)\right)$$

$$-wx_{\max}\frac{d}{dt}\left(\sin(wt)\right)$$

$$-w^2x_{\max}\left(\cos(wt)\right) = -\left(k_{sp}/m\right)x_{\max}\cos(wt)$$

$$w^2 = \left(k_{sp}/m\right)$$

$$w = \sqrt{\left(k_{sp}/m\right)}$$