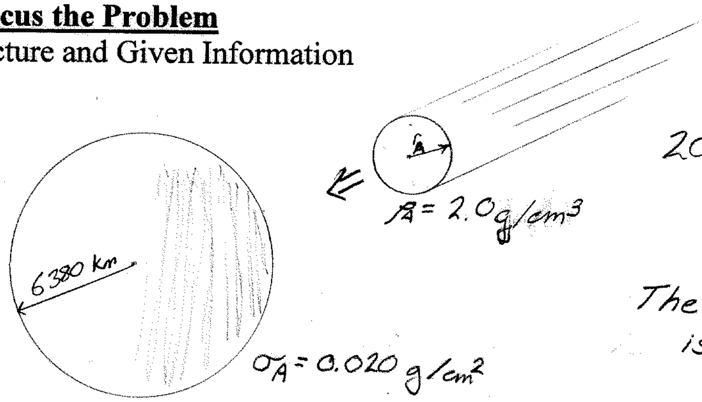
Focus the Problem

Picture and Given Information



20% of asteroid's mass covers the Earth's surface with a density, $\sigma_A = 0.020 g/cm^2$.

The asteroid's density, ρ_A, is $2.0 g/cm^3$.

Question(s)

What radius an asteroid has enough mass so 20% can cover the Earth's surface with $0.020 gm/cm^2$?

Approach

I'll consider geometric relations

Describe the Physics

Diagram(s) and Definite Quantities

$R_E = 6380 km \cdot \frac{1000 m}{1 km} = 6.38 \times 10^6 m$ Earth's radius

$r_A = ?$ Asteroid's radis

$M_A = ?$ Asteroid's mass

$\rho_A = 2.0 g/cm^3 \cdot \frac{1 kg}{1000 g} \cdot \left(\frac{10^3 cm}{1m}\right)^3 = 2.0 \times 10^6 kg/m^3$ Asteroid's density

$V_A = ?$ Asteroids Volume

$\sigma_{Ad} = 0.020 g/cm^2 \cdot \frac{1 kg}{1000 g} \cdot \left(\frac{10^3 cm}{1m}\right)^2 = 20\ kg/m^2$ density of Asteroid dust

$F_{Ad} = 0.2 = \frac{M_{AD}}{M_A}$ fraction of Asteroid that becomes dust

$A_E = ?$ Surface area of Earth

$M_{AD} = ?$ Mass of Asteroid dust on Earth

Target Quantity(ies)

r_A radius of asteroid

Quantitative Relationships

$1 km = 10^5 cm$ $\quad V_{Sphere} = \frac{4}{3}\pi R^3 \Rightarrow V_A = \frac{4}{3}\pi r_A^3$

$\rho = \frac{m}{V} \Rightarrow \rho_A = \frac{m_A}{V_A}$

$A_{Sphere} = 4\pi R^2 \Rightarrow A_E = 4\pi R_E^2$

$\sigma = \frac{m}{A} \Rightarrow \sigma_{Ad} = \frac{m_{Ad}}{A_E}$

$F_{AD} = \frac{M_{AD}}{M_A}$

Plan the Solution
Construct specific equations

Find r_A — Unknowns

$V_A = \frac{4}{3}\pi r_A^3$ — V_A

$r_A = \left(\frac{3}{4\pi} V_A\right)^{\frac{1}{3}}$

Find V_A

$\rho_A = \frac{m_A}{V_A}$ — m_A

$V_A = \frac{m_A}{\rho_A}$

Find m_A

$F_{AD} = \frac{M_{AD}}{M_A}$ — M_{AD}

$M_A = \frac{M_{AD}}{F_{AD}}$

Find M_{AD}

$\sigma_{Ad} = \frac{M_{AD}}{A_E}$ — A_E

$M_{AD} = A_E \cdot \sigma_{Ad}$

Find A_E

$A_E = 4\pi R_E^2$ — none

$M_{AD} = (4\pi R_E^2)\,\sigma_{Ad}$

$M_A = \frac{(4\pi R_E^2 \sigma_{Ad})}{F_{AD}}$

$V_A = \left(\frac{4\pi R_E^2 \sigma_{Ad}}{F_{AD}}\right) / \rho_A$

$r_A = \left(\frac{3}{4\pi}\left(\frac{4\pi R_E^2 \sigma_{Ad}}{F_{AD}\,\rho_A}\right)\right)^{\frac{1}{3}}$

$$\boxed{r_A = \left(\frac{3 R_E^2 \sigma_{AD}}{F_{AD}\,\rho_A}\right)^{\frac{1}{3}}}$$

Check Units

$$[length] = \left(\frac{1\,[length]^2 \frac{[mass]}{[length]^2}}{\frac{1\,[mass]}{[length]^3}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{1}{\frac{1}{[length]^3}}\right)^{\frac{1}{3}}$$

$$= ([length]^3)^{\frac{1}{3}}$$

$$= [length] \checkmark$$

Execute
Calculate target quantity(ies)

$$r_A = \left(\frac{3 \cdot (6.38\times10^6\,m)^3 \cdot 20\,kg/m^2}{0.2 \cdot 2.0\times10^6\,kg/m^3}\right)^{\frac{1}{3}}$$

$$\boxed{r_A = 3.4\times10^5\,m}$$

An asteroid that could reproduce the prehistoric desaster would have to have a radius on the order of 3.4×10^5 m, or about 5% that of the Earth. This is what you must watch for.

Evaluate
Is the answer properly stated? ✓

Is the answer reasonable? nowhere near as large as the Earth, but not insignificant. Seems reasonable.

Is the answer complete? ✓